Effect of Various Misalignments on Static Aeroelastic Characteristics of Guided Launch Vehicles

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Introduction

A MATRIX formulation of the problem of static aeroelastic analysis of guided slender launch vehicles using a discrete analogous system is presented in Ref. 1. The purpose of the present Note is to extend this analysis to include the interstage, fin, and thrust misalignment effects which occur due to mechanical tolerances and to varying fabrication skill. This can be done by simply rewriting Eq. (1) of Ref. 1 as

$$F_r = m_r u\dot{\gamma} + qC_{N_\alpha} S_r(\alpha+\theta_r) + \delta_{rn}P_n + C_r \qquad (r=1,2,...,N) \tag{1}$$

where C_r is the additional term and is the resultant force at the rth station due to various misalignments. With the above basic equation, we obtain the two unknowns P_n and α in the following form (see Ref. 1):

$$P_n = W\left(\frac{x_{cg}-x_{cp}}{x_{cp}-x_n}\right) + q\overline{C_{N_\alpha}S}\left\lfloor\frac{x_r-x_{cp}}{x_{cp}-x_n}\right\rfloor \lfloor Q_r \rfloor \{\theta_r\} + C\left(\frac{x_c-x_{cp}}{x_{cp}-x_n}\right) \tag{2}$$

and

$$\alpha = \frac{W}{q\overline{C_{N_\alpha}S}}\left(\frac{x_n-x_{cg}}{x_{cp}-x_n}\right) + \left\lfloor\frac{x_n-x_r}{x_{cp}-x_n}\right\rfloor \lfloor Q_r \rfloor \{\theta_r\} + \frac{C}{q\overline{C_{N_\alpha}S}}\left(\frac{x_n-x_c}{x_{cp}-x_n}\right) \tag{3}$$

where $Q_r = C_{N_\alpha}S_r/\overline{C_{N_\alpha}S}$; $C=\Sigma_{r=1}^{N}C_r$ is the resultant misalignment force; and $x_c = \Sigma_{r=1}^{N}C_r x_r/C$ is its point of application from the x origin. The third term of Eqs. (2) and (3) represent the effect of misalignment force on the pitch control force and the angle of attack respectively. Also, the final equation for θ_r's is written as

$$\{\theta_r\} = Mu\dot{\gamma}[\rho]\left\{\frac{m_r}{M}\right\} + Mu\dot{\gamma}\left(\frac{x_n-x_{cg}}{x_{cp}-x_n}\right)[\rho]\lfloor Q_r \rfloor\{1\} - Mu\dot{\gamma}\left(\frac{x_{cp}-x_{cg}}{x_{cp}-x_n}\right)[\rho]\{\delta_{rn}\} + q\overline{C_{N_\alpha}S}[\rho]\lfloor Q_r \rfloor\{\theta_r\}$$
$$+ q\overline{C_{N_\alpha}S}[\rho]\{\delta_{rn}\}\left\lfloor\frac{x_r-x_{cp}}{x_{cp}-x_n}\right\rfloor\lfloor Q_r \rfloor\{\theta_r\} - q\overline{C_{N_\alpha}S}[\rho]\lfloor Q_r \rfloor\{1\}\left\lfloor\frac{x_r-x_n}{x_{cp}-x_n}\right\rfloor\lfloor Q_r \rfloor\{\theta_r\} + [\rho]\{C_r\}$$
$$+ C\left(\frac{x_n-x_c}{x_{cp}-x_n}\right)[\rho]\lfloor Q_r \rfloor\{1\} - C\left(\frac{x_{cp}-x_c}{x_{cp}-x_n}\right)[\rho]\{\delta_{rn}\} \tag{4}$$

The last three terms on the right side of Eq. (4) account for the interaction of misalignment effects on the elastic behavior of the vehicle, and they give rise to a fixed angle of attack at various stations as do the inertia forces.

Misalignments

The term C_r in Eq. (1) is the summation of forces due to misalignments at the rth station and can be calculated in the following way. We consider all the misalignment angles to be measured with respect to the first stage axis. If δ_i denotes the misalignment angle of the $(i+1)$st stage, then the aerodynamic force at $x=x_r$ due to this misalignment is given by

$$(C_r)_{\text{int}} = qC_{N_\alpha}S_r\delta_i \qquad x_i < x_r < x_{i+1} \qquad (i=1,2,...,K-1)$$

where K denotes the total number of stages; x_i denotes the distance to the junction of i and $(i+1)$st stage from x origin; and $(C_r)_{\text{int}}$ denotes the aerodynamic force at the rth station due to interstage misalignment. Similarly, the forces due to fin and thrust misalignments can be obtained from the following relations as

$$C_{\text{fin}} = qC_{N_\alpha}S_f\delta_f \qquad C_T = T\delta_T$$

where $C_{N_\alpha}S_f$ is the value of $C_{N_\alpha}S$ of fins; and δ_f and δ_T denote the fin and thrust misalignment angles with respect to the reference axis. It is convenient to take two of N discrete element stations: at the center of pressure of a fin; and at the point where the thrust vector acts, so that C_{fin} and C_T is added to the vector of $(C_r)_{\text{int}}$ at the appropriate stations.

Discrete Representation

Reference 2 described a method of discretization of mass and $(C_{N_\alpha}S)_x$ distribution properties. In that method, concentricity of center of gravity and center of pressure at any discrete element station was achieved by using different station boundaries for mass and $(C_{N_\alpha}S)_x$ distribution properties. Here an analogous method is presented which is convenient for programming. This can be done in the following way. Let i and $i+1$ be two stations where some continuous property, say $f(x)$, is to be discretized. Then the area under the curve between these two stations, A_i, and its center of gravity can be calculated. Let the center of gravity of the area A_i be at a distance $\bar{x}_i$ from the x origin. Now it is assumed that A_i is supported by two reactions at the i and $i+1$ stations. These two reactions will form the discrete values of the property at stations i and $i+1$ due to the $f(x)$ distribution between these stations. The total discrete value f_i of the $f(x)$ property at the ith station is then written as

$$f_i = \left(\frac{x_{i+1}-\bar{x}_i}{x_{i+1}-x_i}\right)A_i + \left(\frac{x_i-\bar{x}_{i-1}}{x_i-x_{i-1}}\right)A_{i-1} \qquad (i=1,2,...,N)$$

At the lower limit of $i(=1)$, $x_{i-1}=0$ is the point where the x origin is taken, and at the upper limit of $i(=N)$, $x_{i+1}=x_{N+1}$ is the distance to the nosecone end from the x origin. This procedure can now be applied for discretization of mass and $(C_{N_\alpha}S)_x$ distribution properties.

This Note essentially complements the formulation presented earlier in Ref. 1 by including the various

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misalignment effects. Also, a simple method of discretization that is suitable for programming is discussed here. At this stage, it is suggested that the entire problem of the static aeroelastic analysis of guided launch vehicles, including various misalignment effects, may be approached systematically in order to find out which factor has the greatest contribution to the aeroelastic behavior of the vehicle. As a first step, one may analyze this problem by including the flexibility of vehicle and assuming rigid joints. Later, joint rotations, misalignment effects, etc., may be included in succession. This will provide a clear picture to the understanding of the problem. Also, it may be of considerable help to the designer before he takes up the dynamic aeroelastic investigations.

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References

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Comparison of Rocket Nozzle Heat Transfer Calculation Methods

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Nomenclature

C = density-viscosity ratio, Eq. (5d)
c_f = friction coefficient $= 2\tau_w/(\rho_e U_e^2)$
D = local diameter of nozzle, m
Ec = Eckert number, Eq. (5j)
F,G,H = functions defined in Eqs. (5g-i)
f_η = $\bar{u}/U_e$
h = specific enthalpy, J/kg
h^0 = specific total enthalpy, J/kg
$\hat{h}^0$ = dimensionless total enthalpy
k = heat conductivity coefficient, W/m-K
M = Mach number
Pr = heat flux, W/m^2
R_θ, R_{δ_H} = Reynolds number based on θ and δ_H, respectively
r = recovery factor, Eq. (2b); local radius from axis of nozzle, m
r_0 = local nozzle wall radius, m
St_r = Stanton number based on recovery temperature
s = transformed coordinate along nozzle wall, Eq. (5b)
T = temperature, K
U_e = freestream gas velocity, m/s
u,v = gas velocity components along and normal to wall, m/s
V = dimensionless normal velocity, Eq. (5a)
x,y = physical coordinates along and normal to wall, m
β = pressure gradient parameter, Eq. (5e)
γ = specific heat ratio
δ^* = displacement thickness $= \int_0^\infty [1-(\rho u/\rho_e U_e)]\,dy$, m
δ_H = enthalpy thickness $= \int_0^\infty (\rho u/\rho_e U_e)(1-\hat{h}^0)\,dy$, m
ϵ = eddy viscosity coefficient, m^2/s
ϵ_h = eddy conductivity coefficient, m^2/s
η = transformed coordinate, Eq. (5c)
θ = momentum thickness, m $= \int_0^\infty (\rho u/\rho_e U_e)[1-(u/U_e)]\,dy$
μ = dynamic viscosity coefficient, kg/m-s
ν = kinematic viscosity coefficient, m^2/s
ψ = stream function, kg/m-s
ρ = mass density, kg/m^3
τ_w = shear stress on wall, N/m^2
ω = viscosity-temperature exponent in relation $\mu \sim T^\omega$

Subscripts and superscripts

$(\bar{\ })$ = time-averaged property
e = inviscid flow property
r = property at recovery temperature or enthalpy
s,η,x,y = derivatives with respect to the coordinate, respectively
w = property at the wall
$(\ )^*$ = property at recovery temperature or enthalpy

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Introduction

HEAT flux and boundary-layer thickness for a rocket nozzle with real-gas properties are compared in the present Note for the purpose of evaluating the applicability of two approximate methods: fully developed turbulent pipe-flow applied to rocket nozzles by Bartz,[1] and an integral method by Bartz.[2] A differential method,[3] which has not been applied previously to realistic rocket nozzle heat transfer calculations, is used here for comparison. Inviscid flow properties for one-dimensional equilibrium flow required for all three methods were computed for a typical solid-propellant rocket of about 75 cm exit diameter and an approximate throat to exit area ratio of 0.2 (Fig. 1). All calculations were done for the nozzle wall temperature of 300 K.

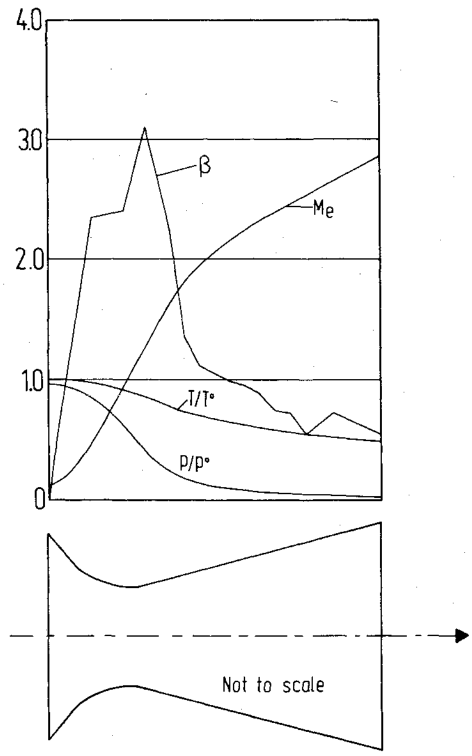

Fig. 1 Inviscid flow properties of a rocket nozzle.